

Determinatlar: fizika, algebra va geometriyada chiziqli tenglamalar sistemasining universal metodi

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Annotatsiya: Mazkur maqolada determinantlar tushunchasi hamda ularning fizika, algebra va geometriya fanlarida qo'llanilishi yoritiladi. Xususan, chiziqli tenglamalar sistemasini yechishda determinantlarning universal metod sifatidagi o'rni tahlil qilinadi. Kramer qoidasi, matritsalar nazariyasi va determinantlarning geometrik hamda fizik ma'nosi misollar orqali tushuntiriladi. Shuningdek, determinantlar yordamida ko'p o'zgaruvchili chiziqli sistemalarni samarali yechish usullari, ularning afzalliklari va amaliy qo'llanilish sohalari ko'rib chiqiladi. Tadqiqot natijalari determinantlar nafaqat algebraik hisoblash vositasi, balki turli fanlararo masalalarni hal etishda muhim universal metod ekanligini ko'rsatadi.

Kalit so'zlar: determinant, matritsa, chiziqli tenglamalar sistemasi, Kramer qoidasi, algebra, geometriya, fizika, vektor, chiziqli algebra, universal metod, ko'p o'zgaruvchili tenglamalar, matematik modellashtirish

Determinants: A Universal Method for Systems of Linear Equations in Physics, Algebra, and Geometry

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Abstract: This article explores the concept of determinants and their application in the fields of physics, algebra, and geometry. Specifically, it analyzes the role of determinants as a universal method for solving systems of linear equations. Cramer's rule, matrix theory, and the geometric and physical interpretations of determinants are explained through examples. Furthermore, the article examines methods for efficiently solving multivariable linear systems using determinants, along with their advantages and areas of practical application. The research findings demonstrate that determinants serve not merely as a tool for algebraic calculation, but as an essential, universal method for addressing a variety of interdisciplinary problems.

Keywords: determinant, matrix, system of linear equations, Cramer's rule, algebra, geometry, physics, vector, linear algebra, universal method, multivariable equations, mathematical modeling

Chiziqli tenglamalar sistemasi ko'plab ilmiy va amaliy masalalarning matematik modeli sifatida namoyon bo'ladi. Bunday sistemalarni yechishning turli usullari mavjud bo'lsa-da, determinantlarga asoslangan usullar, xususan, Kramer qoidasi, o'zining aniqligi, nazariy soddaligi va universalligi bilan ajralib turadi. Determinant yordamida sistemaning yechimga ega yoki ega emasligini aniqlash, yagona yechimni topish hamda geometrik talqinini berish mumkin.

Fizika fanida determinantlar mexanik tizimlar, elektr zanjirlar, kvant jarayonlar va koordinatalar almashinuvi kabi masalalarda uchraydi. Geometriyada esa ular tekislik va fazodagi shakllarning yuzi, hajmi hamda yo'nalishini aniqlashda muhim rol o'ynaydi. Shu sababli determinantlar nafaqat algebraik vosita, balki turli fanlar o'rtasida bog'lovchi universal matematik metod sifatida qaraladi.²

Mazkur maqolaning maqsadi determinant tushunchasining mohiyatini ochib berish, uning fizika, algebra va geometriyadagi qo'llanishlarini yoritish hamda chiziqli tenglamalar sistemasini yechishda universal metod sifatidagi ahamiyatini tahlil qilishdan iborat.

ALGEBRAIK MISOLLARDA QO'LLANILISHI

Kramer usuli haqida ma'lumot.

Kramer qoidasi - chiziqli tenglamalar sistemasini determinantlar yordamida yechish usulidir. Bu usul sistema aniqlovchisi (determinanti) noldan farqli ($A \neq 0$) bo'lgandagina qo'llaniladi.

$$x_1 = \frac{\Delta_1}{\Delta} \quad x_2 = \frac{\Delta_2}{\Delta} \quad x_3 = \frac{\Delta_3}{\Delta}$$

Bu yerda:

Δ - sistemaning asosiy determinanti.

Δ_n - asosiy determinantdagi n-ustunni ozod hadlar ustuni bilan almashtirishdan hosil bo'lgan determinant.

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 4 \\ -1 & 6 & 6 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 4 \\ -1 & 6 & 6 \end{vmatrix} = 0 + 0 - 8 - 0 - 12 - 24 = -44$$

² Determinantning ko'p qirrali tabiati: geometrik hajmdan fizik muvozanatgacha OU Pulatov, M Ashirova - Science and Education, 2026

$$\Delta_x = \begin{vmatrix} 0 & 2 & 0 \\ 10 & 0 & 4 \\ 20 & 6 & 6 \end{vmatrix} = 0 \cdot 40 + 120 - 0 - 0 - 60 = 20$$

$$\Delta_y = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 10 & 4 \\ -1 & 20 & 6 \end{vmatrix} = 60 - 40 + 0 - 0 - 0 - 120 = -100$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 10 \\ -1 & 6 & 20 \end{vmatrix} = 0 + 0 + 0 - 0 - 40 - 40 = -80$$

$$\frac{\Delta_x}{\Delta} = \frac{20}{-44} = -\frac{5}{11} \quad \frac{\Delta_y}{\Delta} = \frac{-100}{-44} = \frac{25}{11} \quad \frac{\Delta_z}{\Delta} = \frac{-80}{-44} = \frac{20}{11}$$

FIZIK MISOLLARDA QO‘LLANILISHI

Kirxgofning birinchi qonuni.

Kramer formulasida toklarni topish:

$$\begin{cases} 1. a_{11} I_1 + a_{12} I_2 + a_{13} I_3 = B_1 \\ 2. a_{21} I_1 + a_{22} I_2 + a_{23} I_3 = B_2 \\ 3. a_{31} I_1 + a_{32} I_2 + a_{33} I_3 = B_3 \end{cases}$$

a - qarshilik, B - EYK

1. Asosiy determinat topiladi

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + a_{13} a_{21} a_{32} + a_{31} a_{12} a_{23} -$$

$$(a_{13} a_{22} a_{31} + a_{11} a_{32} a_{23} + a_{33} a_{21} a_{12})$$

$$\Delta_1 = \begin{vmatrix} B_1 & a_{12} & a_{13} \\ B_2 & a_{22} & a_{23} \\ B_3 & a_{32} & a_{33} \end{vmatrix} = B_1 a_{22} a_{33} + B_2 a_{13} a_{32} + B_3 a_{12} a_{23} -$$

$$(a_{13} a_{22} B_3 + B_1 a_{32} a_{23} + B_2 a_{33} a_{12})$$

$$\Delta_2 = \begin{vmatrix} a_{11} & B_1 & a_{13} \\ a_{21} & B_2 & a_{23} \\ a_{31} & B_3 & a_{33} \end{vmatrix} = a_{11} B_2 a_{33} + a_{13} a_{21} B_3 + a_{31} B_1 a_{23} -$$

$$(a_{13} B_2 a_{31} + a_{11} B_3 a_{23} + a_{33} a_{21} B_1)$$

$$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & B_1 \\ a_{21} & a_{22} & B_2 \\ a_{31} & a_{32} & B_3 \end{vmatrix} = a_{11} a_{22} B_3 + a_{21} a_{32} B_1 + a_{31} a_{12} B_2 -$$

$$(B_1 a_{22} a_{31} + a_{11} a_{32} B_2 + B_3 a_{21} a_{12})$$

$$I_1 = \frac{\Delta_1}{\Delta} \quad I_2 = \frac{\Delta_2}{\Delta} \quad I_3 = \frac{\Delta_3}{\Delta}$$

Tugun uchun (Kirxgof qonuni) I_1 va I_2 birlashib I_3 ni hosil qilmoqda

$I_1 + I_2 - I_3 = 0$ chap kontur uchun (kirxgof II qonuni) soat yo‘nalishi bo‘yicha

$$R_1 \times I_1 + R_3 \times I_3 = E_1$$

O‘ng kontur uchun – soatga teskari yursak

$$R_2 \times I_2 + R_3 \times I_3 = E_2$$

Bularni determinat ko‘rinishiga keltirsak

$$\begin{array}{c}
 I_1 \quad I_2 \quad I_3 \\
 \begin{array}{l}
 E_1 = 10 \text{ V} \\
 E_2 = 20 \text{ V} \\
 R_1 = 2 \Omega \\
 R_2 = 4 \Omega \\
 R_3 = 6 \Omega
 \end{array} \\
 \left| \begin{array}{ccc|c}
 1 & 1 & -1 & 0 \\
 R_1 & 0 & R_3 & I_1 \\
 0 & R_2 & R_3 & E_2
 \end{array} \right| \rightarrow
 \end{array}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 6 \\ 0 & 4 & 6 \end{pmatrix} \times \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 6 \\ 0 & 4 & 6 \end{vmatrix} = 0+0-8-0-12-24 = -44$$

$$\Delta_{x_1} = \begin{vmatrix} 0 & 1 & -1 \\ 10 & 0 & 6 \\ 20 & 4 & 6 \end{vmatrix} = 0-40+120-0-0-60 = 20$$

$$\Delta_{x_2} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 10 & 6 \\ 0 & 20 & 6 \end{vmatrix} = 60-40+0-0-0-120 = -100$$

$$\Delta_{x_3} = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 10 \\ 0 & 4 & 20 \end{vmatrix} = 0+0+0-0-40-40 = -80$$

$$\frac{\Delta_{x_1}}{\Delta} = \frac{20}{-44} = \frac{5}{-11} \quad \frac{\Delta_{x_2}}{\Delta} = \frac{-100}{-44} = \frac{25}{11} \quad \frac{\Delta_{x_3}}{\Delta} = \frac{-80}{-44} = \frac{20}{11}$$

$$\begin{array}{c}
 I_1 \quad R_1 \quad I_1 \quad I_2 \\
 \hline
 \begin{array}{c}
 [E_1] \quad R_3 \quad [E_2]
 \end{array}
 \end{array}$$

GEOMETRIK MASALALARDA QO‘LANILISHI.

Vektorlarni determinant bilan hisoblash.

Agar sizga d va bazis vektorlarning (a, b) koordinatalari berilgan bo'lsa, x, y, z koeffitsiyentlarini topish uchun quyidagi chiziqli tenglamalar sistemasi tuziladi:

$$\begin{cases} a_x x + b_x y + c_x z = d_x \\ a_y x + b_y y + c_y z = d_y \\ a_z x + b_z y + c_z z = d_z \end{cases}$$

Bizda 3ta bazis vektor $\vec{a}, \vec{b}, \vec{c}$ va natijaviy vektor $\vec{d}$ berilgan

$$x, y, z \text{ noma'lum } \vec{d} = x * \vec{a} + y * \vec{b} + z * \vec{c}$$

$$\vec{a}=(1\ 2\ 0), \vec{b}=(1\ 0\ 4), \vec{c}=(-1\ 6\ 6), \vec{d}=(0\ 10\ 20)$$

$$\begin{cases} x+y-z=0 \\ 2x+6z=10 \\ 4y+6z=20 \end{cases}, \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 4 \\ -1 & 6 & 6 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 4 \\ -1 & 6 & 6 \end{vmatrix} = 0+0-8-0-12-24 = -44$$

$$\Delta_x = \begin{vmatrix} 0 & 2 & 0 \\ 10 & 0 & 4 \\ 20 & 6 & 6 \end{vmatrix} = 0-40+120-0-0-60 = 20$$

$$\Delta_y = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 10 & 4 \\ -1 & 20 & 6 \end{vmatrix} = 60-40+0-0-0-120 = -100$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 10 \\ -1 & 6 & 20 \end{vmatrix} = 0+0+0-0-40-40 = -80$$

$$\frac{\Delta_x}{\Delta} = \frac{20}{-44} = \frac{5}{-11}, \frac{\Delta_y}{\Delta} = \frac{-100}{-44} = \frac{25}{11}, \frac{\Delta_z}{\Delta} = \frac{-80}{-44} = \frac{20}{11}$$

$$\vec{d} = \frac{5}{-11} \vec{a} + \frac{25}{11} \vec{b} + \frac{20}{11} \vec{c}$$

Xulosa:

Mazkur maqolada determinantlar tushunchasi hamda ularning fizika, algebra va geometriya fanlaridagi o'rni va ahamiyati tahlil qilindi. Determinantlar chiziqli tenglamalar sistemasini yechishda muhim matematik vosita bo'lib, ayniqsa Kramer qoidasi orqali ko'p o'zgaruvchili sistemalarning yechimini aniqlashda universal metod sifatida xizmat qilishi ko'rsatildi. Shuningdek, determinant yordamida sistemaning yechimga ega yoki ega emasligini aniqlash, yagona yechimni topish hamda matematik jarayonlarni soddalashtirish mumkinligi asoslab berildi.

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