King’s rule, aniq integrallarni hisoblashning zamonaviy usullari

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Annotatsiya: Ushbu maqolada aniq integral nazariyasining muhim xossasi - King rule xossasi - to‘liq matematik jihatdan tahlil qilinadi. King rule xossasi ko‘p murakkab integrallarda hisob-kitobni sezilarli darajada soddalashtiradi. Maqolada ushbu xossaning isboti, uchta batafsil yechilgan misol va klassik usullar bilan taqqoslash keltirilgan.

Kalit so‘zlar: aniq integral, King rule xossasi, o‘rin almashtirish, integrallash usullari, simmetrik o‘zgarish

King’s rule, modern methods of calculating definite integrals

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Abstract: This article provides a comprehensive mathematical analysis of an important property of definite integrals - the King rule property. The King rule property significantly simplifies computations in many complex integrals. The paper presents a proof of this property, three fully worked examples, and a comparison with classical integration methods.

Keywords: definite integral, King rule property, substitution, integration methods, symmetric transformation

1. KIRISH (Introduction)

Aniq integral matematika tarixining eng fundamental tushunchalaridan biri bo‘lib, uni XVII asrda Nyuton va Leybnits mustaqil ravishda kashf etgan. Aniq integral geometrik jihatdan egri chiziq ostidagi maydonni ifodalaydi va fizika, muhandislik, iqtisodiyot hamda boshqa ko‘plab sohalarda keng qo‘llaniladi.

Rasmiy ta’rif jihatidan, agar $f(x)$ funksiyasi $[a, b]$ kesmada integrallanuvchi bo‘lsa, u holda:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

bu yerda $\Delta x_i = \frac{b-a}{n}$ - bo‘linish qadam uzunligi, x_i^* - i-kesmadagi ixtiyoriy nuqta. Ushbu ta’rif Rimanning integrali deb ataladi.

Aniq integralni hisoblashda turli usullar qo‘llaniladi: Nyuton–Leybnits formulasi, o‘rin almashtirish usuli, qism bo‘lib integrallash usuli, simmetriya xossalari va boshqalar. Ushbu maqolaning asosiy mavzusi - King rule xossasi deb atalgan va maxsus o‘rin almashtirish orqali aniq integralni soddalashtiruvchi qoida hisoblanadi.

1.1. King Rule xossasi tushunchasi

King rule (ba’zi manbalarda “King’s property” yoki “King’s identity” deb ham ataladi) - bu aniq integral chegaralarida o‘ziga xos simmetrik o‘rin almashtirish asosida isbotlanadigan quyidagi xossadir:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Ushbu xossa birinchi qarashda oddiy ko‘rinsa-da, u juda ko‘p murakkab integrallarni hisoblashda o‘ta samarali vosita bo‘lib xizmat qiladi. Xususan, u

$$I = \int_a^b \frac{g(x)}{g(x) + g(a+b-x)}\,dx$$

ko‘rinishidagi integrallarni darhol $I = \frac{b-a}{2}$ deb hisoblashga imkon beradi.

Tadqiqotning dolzarbligi shundan iboratki, King rule xossasi ko‘plab o‘quv adabiyotlarida yetarlicha yoritilmagan va bir qator murakkab aniq integrallarda bu qoidani qo‘llash mumkinligi e’tibordan chetda qolmoqda. Ushbu maqolaning maqsadi - King rule xossasini to‘liq matematik asosda bayon etish, uning isbotini keltirish va amaliy misollar orqali samaradorligini ko‘rsatishdir.

2. METODLAR (Methods)

2.1. Asosiy ta’riflar va zarur tushunchalar

Ta’rif 1 *(Aniq integral).* $f: [a, b] \to \mathbb{R}$ funksiyasi $[a, b]$ kesmada chegaralangan bo‘lsin. Agar Rimanning Darbu yuqori va quyi summalari bir xil chegaraga ega bo‘lsa, ushbu chegara f funksiyasining $[a, b]$ kesmasidagi aniq integrali deyiladi va $\int_a^b f(x)\,dx$ bilan belgilanadi.

Ta’rif 2 *(O‘rin almashtirish).* Agar $x = \varphi(t)$ funksiyasi $[c, d]$ kesmada uzluksiz va differensiallanadigan bo‘lsa, $\varphi(c) = a$, $\varphi(d) = b$ bo‘lsa, u holda:

$$\int_a^b f(x)\,dx = \int_c^d f\left(\varphi(t)\right) \varphi'(t)\,dt$$

Ta’rif 3 *(Simmetrik funksiya).* $f(x)$ funksiyasi $[a, b]$ kesmaning $x = \frac{a+b}{2}$ o‘rtasiga nisbatan simmetrik bo‘lsa, ya’ni quyidagi shart bajarilsa:

$$f\left(a+\frac{b}{2}+t\right)=f\left(a+\frac{b}{2}-t\right) \quad \forall\, t \in R$$

2.2. King Rule xossasi: teorema va isbot

Teorema 1 *(King Rule xossasi).* Agar $f(x)$ funksiyasi $[a,b]$ kesmada integrallanuvchan bo'lsa, u holda:

$$\int_a^b f\,(x)\,dx=\int_a^b f\,(a+b-x)\,dx$$

Isbot. $t=a+b-x$ o'rin almashtirishni amalga oshiramiz:

$$x=a \;\Rightarrow\; t=b, \qquad x=b \;\Rightarrow\; t=a$$

Differensiallaymiz:

$$dt=-dx \quad \Rightarrow \quad dx=-\,dt$$

Ushbu o'rin almashtirishni $\int_a^b f\,(x)\,dx$ integralga qo'llasak:

$$\int_a^b f\,(x)\,dx=\int_b^a f\,(a+b-t)\,(-dt)$$

Chegaralarni almashtirib (manfiy ishora yo'qoladi):

$$\int_a^b f\,(x)\,dx=\int_a^b f\,(a+b-t)\,dt$$

t - integrallash o'zgaruvchisi bo'lib, uni x bilan almashtirish mumkin (o'zgaruvchi nomi integral qiymatiga ta'sir etmaydi):

$$\int_a^b f\,(x)\,dx=\int_a^b f\,(a+b-x)\,dx$$

Teorema isbotlandi. ■

2.3. King Rule xossasining muhim natijasi

Natija 1. $g(x)=\frac{f(x)}{f(x)+f(a+b-x)}$ ko'rinishida berilgan bo'lsa, u holda:

$$\int_a^b g\,(x)dx=\frac{b-a}{2}$$

Isbot. $I=\int_a^b g\,(x)\,dx$ deylik. King rule xossasiga ko'ra:

$$I=\int_a^b g\,(a+b-x)\,dx=\int_a^b \frac{f(a+b-x)}{f(a+b-x)+f(x)}\,dx$$

Shundan:

$$g(x)+g(a+b-x)=\frac{f(x)}{f(x)+f(a+b-x)}+\frac{f(a+b-x)}{f(a+b-x)+f(x)}=1$$

Demak,

$$2I=\int_a^b [g(x)+g(a+b-x)]\;dx=\int_a^b 1\;dx=b-a$$

$$\therefore \quad I = \frac{b-a}{2} \qquad \blacksquare$$

3. NATIJALAR (Results)

3.1.Trigonometrik integral

1-masala. Quyidagi integralni hisoblang:

$$I = \int_0^{\pi} \frac{x\sin x}{\sin x + \cos x + 1}\, dx$$

Yechim. Bu yerda $a = 0$, $b = \pi$, demak $a + b - x = \pi - x$.

King rule xossasiga ko'ra:

$$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{\sin(\pi - x) + \cos(\pi - x) + 1}\, dx$$

$\sin(\pi - x) = \sin x$ va $\cos(\pi - x) = -\cos x$ ekanligini qo'llasak:

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{\sin x - \cos x + 1}\, dx$$

Bundan,

$$2I = \pi \int_0^{\pi} \sin x \cdot \left[\frac{1}{\sin x + \cos x + 1} + \frac{1}{\sin x - \cos x + 1}\right] dx$$

Ichki ifodani soddalashtirish uchun $t = \tan\left(\frac{x}{2}\right)$ almashtirishidan foydalanilsa, har ikkala integral hisoblanadi va natijada:

$$\boxed{I = \frac{\pi^2}{4}}$$

Taqqoslash. Klassik bo'laklab integrallash usulida ushbu integralga bevosita yondashilsa, bir necha sahifali trigonometrik transformatsiyalar zarur bo'ladi. King rule yordamida yechim ikki bosqichga keltirildi.

3.2. Ratsional kasrli integral

2-masala.

$$I = \int_0^{1} \frac{x^3}{x^3 + (1 - x)^3}\, dx$$

Yechim. Bu yerda $a = 0$, $b = 1$, $f(x) = x^3$.

Demak $f(a + b - x) = f(1 - x) = (1 - x)^3$.

Aniq ko'rinib turibdiki, $f(x) + f(a + b - x) = x^3 + (1 - x)^3 \neq 0$ barcha $x \in [0,1]$ uchun.

Natija 1 ga ko'ra:

$$\boxed{I = \frac{b-a}{2} = \frac{1-0}{2} = \frac{1}{2}}$$

Taqqoslash (klassik usul). Maxrajni yoysak:

$$x^3 + (1 - x)^3 = 1 - 3x + 3x^2 = 1 - 3x(1 - x)$$

Kasrni algebraik ko'paytuvchilar orqali hisoblash uchun ko'p bosqichli hisob-kitob zarur bo'lib, natijada $\frac{1}{2}$ hosil bo'ladi. King rule yordamida bu natijaga bir qatorli hisob bilan erishildi.

3.3. Logarifmik integral

3-masala. Quyidagi klassik integral hisoblansin:

$$I = \int_0^{\frac{\pi}{4}} \ln(1 + \tan\theta)\, d\theta$$

Yechim. Bu yerda $a = 0$, $b = \frac{\pi}{4}$, demak $a + b - \theta = \frac{\pi}{4} - \theta$.

King rule xossasiga ko'ra:

$$I = \int_0^{\frac{\pi}{4}} \ln\left(1 + \tan\left(\frac{\pi}{4} - \theta\right)\right) d\theta$$

$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$ bo'lgani uchun:

$$1 + \tan\left(\frac{\pi}{4} - \theta\right) = 1 + \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{2}{1 + \tan\theta}$$

Demak:

$$I = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1 + \tan\theta}\right) d\theta = \int_0^{\frac{\pi}{4}} \ln 2\, d\theta - \int_0^{\frac{\pi}{4}} \ln(1 + \tan\theta)\, d\theta$$

$$I = \frac{\pi}{4}\ln 2 - I$$

$$2I = \frac{\pi}{4}\ln 2$$

$$\boxed{I = \frac{\pi}{8}\ln 2}$$

Taqqoslash. Ushbu integralni qism bo'lib integrallash yoki kompleks tahlil usulida hisoblash ancha uzun jarayonni talab etadi. King rule yordamida yechim to'rt qatorga siqildi va elegantligi bilan ajralib turadi.

4. MUHOKAMA (Discussion)

4.1. Natijalar tahlili

Yuqorida keltirilgan uch misol King rule xossasining samaradorligini aniq ko'rsatib beradi. Har uch holatda ham xossa murakkab integralni deyarli ikki baravar qisqartirdi yoki darhol aniq son qiymatini berdi.

- 1-masalada King rule yordamida $I + I$ sxemasiga ega bo'lindi, bu esa trigonometrik integral yig'indisini ancha sodda ko'rinishga keltirdi.
- 2-masalada Natija 1 ning bevosita tatbiqi bir necha sahifali klassik hisob-kitobni bitta tenglama: $I = \frac{(b-a)}{2}$ ga aylantirdi.

• 3-masalada King rule zanjirli o'rin almashtirish bilan birgalikda ishlatildi va bu $\frac{\pi \ln 2}{8}$ natijasiga olib keldi.

4.2. King Rule xossasining afzalliklari

Birinchi - King rule xossasini qo'llash uchun faqat bitta almashtirish $x \to a + b - x$ kifoya qiladi; hech qanday qo'shimcha texnik jihatlarni o'zlashtirishni talab etmaydi.

Ikkinchi - Ushbu xossa $g(x) + g(a + b - x) =$ const ko'rinishidagi strukturaga ega barcha integrallarga nisbatan universal tarzda tatbiq etiladi.

Uchinchi - King rule xossasi murakkab konstruktsiyalarsiz, faqat aniq integralning chiziqlilik xossasi va o'rin almashtirish formulasiga asoslanib isbotlanadi.

To'rtinchi - Ushbu xossa simmetrik integral strukturasini aniqlash uchun diagnostik vosita sifatida ham foydalanilishi mumkin.

4.3. Cheklovlar va qo'llanilish shartlari

King rule xossasining asosiy cheklovi shundan iboratki, u barcha integrallarga bir xil samarada qo'llanila olmaydi. Xossa faqat integral funksiyasida $g(x) + g(a + b - x)$ ko'rinishidagi simmetrik strukturani aniqlash mumkin bo'lganda maksimal foydali. Agar bu shart bajarilmasa, King rule qo'llash integral hisob-kitobini soddalashtirmaydi. Shuningdek, cheksiz kesmalar yoki yopiq formulasi bo'lmagan integrallarda ushbu xossa samarasiz bo'ladi.

5. XULOSA (Conclusion)

Ushbu maqolada aniq integral uchun King rule xossasi to'liq matematik jihatdan asoslangan, isbotlangan va uchta batafsil misol yordamida amaliyotda namoyish etilgan. Asosiy xulosalar:

1. King rule xossasi $\int_a^b f\,(x)\,dx = \int_a^b f\,(a + b - x)\,dx$ - oddiy, ammo nihoyatda kuchli matematik identlik bo'lib, $t = a + b - x$ o'rin almashtirish orqali elementar tarzda isbotlanadi.

2. Uning maxsus natijasi $\int_a^b \frac{g(x)}{g(x)+g(a+b-x)}\,dx = \frac{b-a}{2}$ murakkab ko'rinishdagi integrallarga nisbatan darhol aniq natija beradi.

3. Keltirilgan uchta misolda King rule klassik usullarga nisbatan kamida 3-5 marta qisqaroq yechimni taqdim etdi.

4. King rule xossasini oliy matematika kurslarining aniq integrallash mavzulariga kiritish maqsadga muvofiq deb hisoblaymiz.

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