Ba'zi birinchi tartibli oddiy differensial tenglamalar va ularni Maple paketidan foydalanib umumiy yechimini topish

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Annotatsiya: Mazkur maqolada hosilaga nisbatan yehilgan birinchi tartibli differensial tenglama Rikkati tenglamasi, hosilaga nisbatan yechilmagan ba'zi to'la bo'lmagan differensial tenglamar haqida ma'lumot berib ularga oid misollarni Maple paketidan foydalanib umumiy yechimini topishga oid misollar keltirib o'tilgan.

Kalit so'zlar: Rikkati tenglamasi, Maple paketi, to'liq bo'lmagan, yechim

Some first-order ordinary differential equations and finding their general solution using the Maple package

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Abstract: This article provides information about the first-order differential equation Riccati equation solved with respect to the derivative, some incomplete differential equations that are not solved with respect to the derivative, and gives examples of finding their general solution using the Maple package.

Keywords: Riccati equation, Maple package, incomplete, solution

Rikkati tenglamasining umumiy ko'rinishi

$$\frac{dy}{dx} = P(x)y^2 + Q(x)y + R(x) \tag{1}$$

dan iborat. Bunda $P(x) \cdot R(x) \neq 0$ va $P(x), Q(x), R(x)$ lar ko'rilayotgan oraliqda aniqlangan va uzluksiz funksiyalardir

Teorema. Agar Rikkati tenglamasining bitta $y_1(x)$ xususiy yechimi berilgan bo‘lsa, uning umumiy yechimi ikkita kvadratura yordamida aniqlanadi.

Isbot. $y_1(x)$ (1) tenglamaning yechimi bo‘lsin ya’ni

$$y_1' \equiv P(x)y_1^2 + Q(x)y_1 + R(x) \quad (2)$$

$$y = y_1 + \frac{1}{u} \quad (3)$$

almashtirishini olamiz.

$u(x)$ yangi noma’lum funksiya. Bu almashtirish Rikkati tenglamasining chiziqli tenglamaga aylantiradi.

Haqiqatan ham (3) dan.

$$\frac{dy}{dx} = y' = y_1' - \frac{1}{u^2}\frac{du}{dx} \quad (4)$$

(3)va (4) ga asosan (1) tenglamani

$$y_1' - \frac{1}{u^2}\frac{du}{dx} = P(x)\left(y_1 + \frac{1}{u}\right)^2 + Q(x)\left(y_1 + \frac{1}{u}\right) + R(x)$$

$$y_1' - \frac{1}{u^2}\frac{du}{dx} = P(x)y_1^2 + Q(x)y_1 + R(x) + 2P(x)\cdot\frac{y_1}{u} + P(x)\frac{1}{u^2} + Q(x)\cdot\frac{1}{u}$$

(2) ni e’tiborga olsak, keyingi tenglamani

$$\frac{du}{dx} + (2Py_1 + Q)u = -P \quad (5)$$

ko‘rinishga keltirish mumkin. Bu esa chiziqli differensial tenglama bo‘lib uning umumiy yechimi ikkita kvadratura yordamida aniqlanadi.

(5) tenglamadan aniqlangan $u(x)$ qiymatini (3) ga olib borib qo‘ysak, Rikkati tenglamasining umumiy yechimiga ega bo‘lamiz.

Hosilaga nisbatan oshkor bo‘lmagan birinchi tartibli differensial tenglamalar

$$F(x, y', y) = 0 \quad (6)$$

ko’rinishda yoziladi.

(6) tenglamada noma’lum funksiya qatnashmagan tenglamani qaraymiz:

$$F(y', x) = 0 \quad (7)$$

1) Aytaylik (6) tenglamani y' ga nisbatan yechish mumkin deb:

$$y' = f_n(x), \ (n = 1, 2, 3, \ldots, m) \quad (8)$$

Tenglamani hosil qilamiz, (8) ni integrallab $y = \int f_n(x)dx + C, \ (n = 1, 2, 3, \ldots, m)$ ni hosil qilamiz. Bu yechimlar to‘plamiga (8) tenglamaning umumiy integrali deyiladi.

2) Faraz qilaylik, (7) tenglama x ga nisbatan yechilgan deb, ya’ni

$$x = \varphi(y') \quad (9)$$

bo‘lsin. U holda $y' = p$, p - parametr kiritib,

$$x = \varphi(p) \quad (10)$$

ni topamiz. Endi izlanuvchi funksiya y ni ham p orqali ifodalashga kirishamiz. Shu maqsadda asosiy munosabatdan foydalanamiz: $dy = y'dx$ dan $dy = p\varphi'(p)dp$ hosil bo‘ldi. Oxirgi tenglikni ikkala tomonini integrallab $y = \int p\varphi'(p)dp + c$

ga ega bo‘lamiz.

Shunday qilib (9) dastlabki tenglamaning parametrik ko‘rinishdagi umumiy yechimini ushbu ko‘rinishda hosil qildik:

$$x = \varphi(p), \quad y = \int p\varphi'(p)dp + c$$

Izoh: Ba’zi hollarda parametr p ni $p = p(y')$ belgilash orqali kiritish qulay bo‘ladi,

3) Aytaylik (7) tenglamani x ga yoki y' ga nisbatan yechish mumkin bo‘lmasin, faraz qilaylik, (7) tenglamani parametr shaklda yozish mumkin deb olamiz:

$$x = \varphi(t), y' = \psi(t) \quad (11)$$

ya’ni $F[\varphi(t), \psi(t)] = 0$ bajarilsin. Unda asosiy munosabat $dy = y'dx$ dan $dy = \psi(t)\varphi'(t)dt$ kelib chiqadi, oxirgi tenglikni integrallab:

$$y = \int \psi(t)\varphi'(t)dt + c.$$

ni topamiz

Shunday qilib dastlabki tenglamaning umumiy yechimini

$$\begin{cases} y = \int \psi(t)\varphi'(t)dt + c \\ x = \varphi(t) \end{cases}$$

parametrik ko‘rinishda aniqladik. Bunda t - parametr, c - ixtiyoriy o‘zgarmas.

1-misol. Quyidagi Rikkati tenglamasini $y(0) = 1$ shartni qanoatlantiruvchi yechimini toping.

$y' = y^2 - y\sin x + \cos x.$

Yechish. Berilgan tenglamani Maple paketiga kiritamiz:

```
restart;
with(plots):
ode1:= diff(y(x), x)-y(x)^2+y(x)*sin(x)-cos(x);
```

$$ode1 := \frac{d}{dx} y(x) - y(x)^2 + y(x)\sin(x) - \cos(x)$$

DEtools qismpaketidagi odeadvisor funksiyasi yordamida berilgan differensial tenglamaning tipini aniqlaymiz:

```
with(DEtools): odeadvisor(ode1);
```

[_Riccati]

Berilgan differensial tenglama Rikkati tenglamasi ekanligini bildiradi.
Dsolve funksiyasi yordamida tenglamani yechamiz
dsolve({ode1},y(x));
Endi boshlang'ich shartni kiritamiz:

$$\left\{y(x) = -\frac{\exp(-\cos(x))}{c_1 + Int(\exp(-\cos(x)), x)} + \sin(x)\right\}$$

cond:=y(0)=1;

$$cond := y(0) = 1$$

Boshlang'ich shartni qanoatlantiruvchi yechim quyidagicha:
dsolve({ode1,cond},y(x));

$$y(x) = \frac{\exp(-\cos(x))}{\exp(-1) - (Int(\exp(-\cos(\_z1)), \_z1 = 0..x))} + \sin(x)$$

dsolve({ode1,cond},y(x), series);

$$y(x) = 1 + 2x + \frac{3}{2}x^2 + \frac{3}{2}x^3 + \frac{23}{12}x^4 + \frac{263}{120}x^5 + O(x^6)$$

2-misol. $x(y'^2 - 1) = 2y'$ tenglamani yeching.
Yechish. Berilgan tenglamani Maple paketiga kiritamiz:
restart;
Parametr kiritamiz
p := 'p':
x ni p orqali ifodalash
x := 2*p/(p^2 - 1):
$\frac{dx}{dp}$ ni hisoblash
dxdp := diff(x, p):
y ni topish:
y := int(p * dxdp, p):
parametrik ko'rinishidagi yechim
x_p := x;

$$x\_p := \frac{2p}{p^2 - 1}$$

y_p := simplify(y);

$$y\_p := -\frac{1}{p+1} - \ln(p+1) + \frac{1}{p-1} - \ln(p-1)$$

x_p, y_p;

$$\frac{2p}{p^2 - 1}, -\frac{1}{p+1} - \ln(p+1) + \frac{1}{p-1} - \ln(p-1)$$

Foydalanilgan adabiyotlar

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