Eyler integrallari va ularning amaliy tatbiqlari

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Annotatsiya: Mazkur maqolada Eyler tomonidan kiritilgan Beta funksiyasining ta'rifi hamda ular orasidagi bog'lanish keltiriladi. Shuningdek, Eyler integrallaridan foydalanmasdan hisoblash birmuncha murakkab yoki imkonsiz bo'lgan integrallarni Beta funksiyasi yordamida osongina hisoblash mumkinligi ko'rsatiladi. Bu orqali akademik-litsey va oliy ta'lim muassasalari talabalari orasida mavzuga bo'lgan qiziqish orttirish va Eyler integrallarining imkoniyatlarini namoyish etish maqsad qilingan.

Kalit so'zlar: Eyler integrallari, Beta funksiya, integral

Euler integrals and their practical applications

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Abstract: In this paper, the definitions of the Beta functions introduced by Euler and the relationship between them are presented. Furthermore, it is shown that integrals which are rather difficult or impossible to compute without using Euler integrals can be easily evaluated using the Beta functions. This aims to increase the interest of students in academic lyceums and higher education institutions in this topic and to demonstrate the capabilities of Euler integrals.

Keywords: functional equation, mathematical analysis, equation solution, educational process

Ushbu

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx \ (1)$$

Xosmas integral berilgan bo'lsin.

1-ta'rif (1) integral beta funksiya yoki birinchi tur Eylar integrali deyiladi va B(a,b) kabi belgilanadi, demak

$$B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}dx\ (a>0, b>0)$$

Shunday qilib , B(a,b) funksiya R^2 fazodagi M={(a,b)ЄR^2 : aЄ(0; +∞), bЄ(0; +∞)} to'plamda berilgandir. Endi B(a, b) funksiyaning xossalarini ko'rib chiqamiz. B(a, b) integralni olamiz.

$$B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}dx\ (2)$$

Integral a va b ga nisbatan simmetrik funksiya hisoblanadi va quyidagi tenglik o'rinli

$$B(a, b) = B(b, a)\ (3)$$

Isbot. Haqiqatda, $B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}dx$ integralda x=1-t almashtirish bajarilsa, u holda quyidagiga ega bo'lishihi topamiz.

$$B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}dx = -\int_1^0 (1-t)^{a-1}\,t^{b-1}dt = \int_0^1 t^{b-1}\,(1-t)^{a-1}dt = B(b,a)$$

Bunda faqat a va b ning o'rinlari almashadi

(1) integral

$$B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}dx$$

Ixtiyoriy M={(a,b)ЄR^2 : aЄ(0; +∞), bЄ(0; +∞)} (a_0>0, b0>0) to'plamda tekis yaqinlashuvchi bo'ladi.

Isbot. Berilgan integralni tekis yaqinlashuvchilikka tekshirish uchun uni quyidagicha

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx = \int_0^{1/2} x^{a-1}(1-x)^{b-1}dx + \int_{1/2}^1 x^{a-1}(1-x)^{b-1}dx$$

yozib olamiz.

Ma'lumki, a>0 bo'lganda $\int_0^{1/2} x^{a-1}\,dx$ integral yaqinlashuvchi, b>0 bo'lganda $\int_{1/2}^1 (1-x)^{b-1}dx$

integral yaqinlashuvchi bo'ladi.

Parametr a ning a≥a_0 (a_0>0) qiymatlari va ixtiyoriy b>0, ixtiyoriy xЄ(0, 1/2) uchun

$$x^{a-1}(1-x)^{b-1} \le x^{a_0-1}(1-x)^{b-1} \le 2x^{a_0-1}$$

bo'ladi. Veyershtrass alomatidan foydalanib,

$$\int_0^{1/2} x^{a-1}(1-x)^{b-1}dx$$

Integralning tekis yaqinlashuvchiligini topamiz.

Shuningdek parametr b ninng $b \geq b_0$, ($b_0>0$) qiymatlari va ixtiyoriy a>0, ixtiyoriy x$\in$[1/2, 1) uchun,

$$x^{a-1}(1-x)^{b-1} \leq x^{a-1}(1-x)^{b_0-1} \leq 2(1-x)^{b_0-1}$$

bo’ladi va yana Veyershtrass alomatiga ko’ra,

$$\int_{1/2}^{1} x^{a-1}(1-x)^{b-1}dx$$

Xulosa

Beta funksiya (I tur Eyler integrali) matematik tahlilning eng kuchli tahliliy vositalaridan biri bo‘lib, quyidagi xulosalarga kelish imkonini beradi:

Hisoblash samaradorligi: Eyler tomonidan kiritilgan Beta va Gamma funksiyalari oddiy algebraik usullar yoki elementar funksiyalar yordamida hisoblash qiyin yoki mutlaqo imkonsiz bo‘lgan murakkab integrallarni sodda ko'rinishga keltirish va yechish imkonini beradi.

Amaliy ahamiyati: Ushbu integrallar nafaqat nazariy matematikada, balki ehtimollar nazariyasi, fizika va muhandislik kabi sohalarda uchraydigan xosmas integrallarni modellashtirishda muhim ahamiyatga ega.

Foydalanilgan adabiyotlar

1. Azlarov, O. Matematikaning zamonaviy usullari. Toshkent, 2010.
2. Fixtingols, A. Matematik tahlil asoslari. Toshkent, 2012.
3. Sharipov, R. Integral va hosilalar nazariyasi. Toshkent, 2015.
4. Gaziyev, M. Funksional tenglamalar. Toshkent, 2018.
5. Sadullayev, S. Matematika va amaliy fanlar darsligi. Toshkent, 2020.